YEAH Assignment 3

Recursion!

• This assignment is broken down into **4 parts**. We think they're a little more involved than your previous assignments, so be sure to start early!

- This assignment is broken down into **4 parts**. We think they're a little more involved than your previous assignments, so be sure to start early!
- Pair programming is allowed on this assignment!

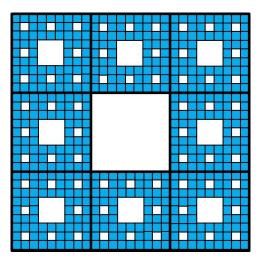
- This assignment is broken down into **4 parts**. We think they're a little more involved than your previous assignments, so be sure to start early!
- Pair programming is allowed on this assignment!
 - Be aware that you should be working together on **all** parts of this assignment. If you don't implement parts of this assignment, you'll be at a significant **disadvantage** on the exams!

- This assignment is broken down into **4 parts.** We think they're a little more involved than your previous assignments, so be sure to start early!
- Pair programming is allowed on this assignment!
 - Be aware that you should be working together on **all** parts of this assignment. If you don't implement parts of this assignment, you'll be at a significant **disadvantage** on the exams!
- Small point: if you're on Windows and you're getting build errors right off the bat, you'll need to redownload the starter code!

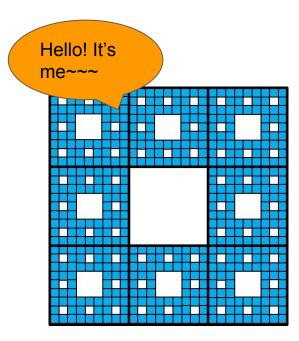
Let's begin!

Remember our beautiful Sierpinski Carpet?

Remember our beautiful Sierpinski Carpet?



Remember our beautiful Sierpinski Carpet?



- Remember our beautiful Sierpinski Carpet?
- We are beginning A3 with another fascinating discovery of Waclaw Sierpinski



- Remember our beautiful Sierpinski Carpet?
- We are beginning A3 with another fascinating discovery of Waclaw Sierpinski

Waclaw Sierpinski, trusted distributor of CS 106B material



Remember our beautiful Sierpinski Carpet?

We are beginning A3 with another fascinating discovery of Waclaw

Sierpinski

"I dedicate these shapes to Keith Schwarz" -Sierpinski, probably

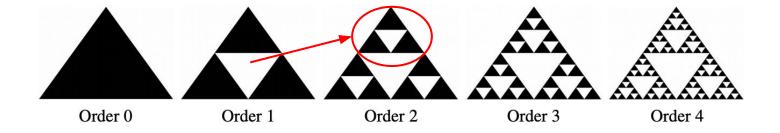
> Waclaw Sierpinski, trusted distributor of CS 106B material











You are writing a program that draws n-order Sierpinski Triangles



- You are writing a program that draws n-order Sierpinski Triangles
- The Sierpinski triangle is defined recursively, meaning:
 - An order-0 Sierpinski triangle is a plain filled triangle.
 - \circ An order-*n* Sierpinski triangle, where n > 0, consists of three Sierpinski triangles of order n 1, each half as large as the main triangle, arranged so that they meet corner-to-corner.





- You are responsible for handling two functions
- The first function draws a black triangle on the canvas given the 3 vertices



- You are responsible for handling two functions
- The first function draws a black triangle on the canvas given the 3 vertices

```
void drawTriangle(GWindow& window,
double x0, double y0,
double x1, double y1,
double x2, double y2);
```



- You are responsible for handling two functions
- The first function draws a black triangle on the canvas given the 3 vertices
- The second function is the recursive function you need to implement



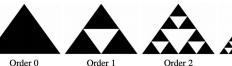
- A few implementation thoughts:
 - o If order is negative you should throw an error!

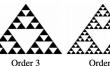


- A few implementation thoughts:
 - o If the order is negative you should throw an error!
 - For any given recursive case, how many calls to drawSierpinskiTriangle() should you be making? At what locations?



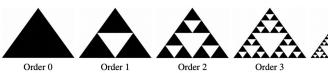
- A few implementation thoughts:
 - If the order is negative you should throw an error!
 - For any given recursive case, how many calls to drawSierpinskiTriangle() should you be making? At what locations?
 - An order-0 Sierpinski triangle is a plain filled triangle.





Order 1 Order 2 Order 3

- A few implementation thoughts:
 - o If the order is negative you should throw an error!
 - For any given recursive case, how many calls to drawSierpinskiTriangle() should you be making? At what locations?
 - An order-0 Sierpinski triangle is a plain filled triangle.
 - \circ An order-n Sierpinski triangle, where n > 0, consists of three Sierpinski triangles of order n
 - 1, each half as large as the main triangle, arranged so that they meet corner-to-corner



A few implementation thoughts:

- If the order is negative you should throw an error!
- For any given recursive case, how many calls to drawSierpinskiTriangle() should you be making? At what locations?
- An order-0 Sierpinski triangle is a plain filled triangle.
- An order-n Sierpinski triangle, where n > 0, consists of three Sierpinski triangles of order n
 1, each half as large as the main triangle, arranged so that they meet corner-to-corner
- Highly recommend drawing this one out and planning exactly where your points are going to be before coding. Even if you understand this problem, it's still easy to make math errors (trust me! - Trip, who gets this wrong every time he tries it)

Questions about Part 1?



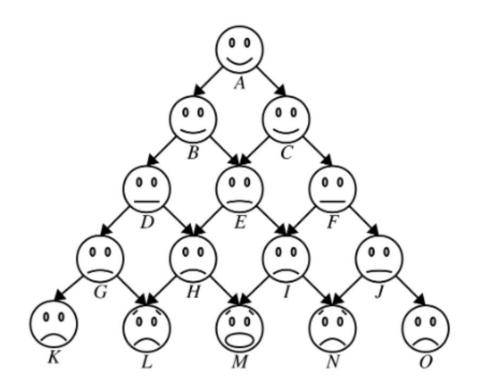
We will now use recursion to examine why life is just not fair...

- We will now use recursion to examine why life is JUST not fair...
- Have you ever made a human pyramid with your friends, and you were placed at the bottom center?

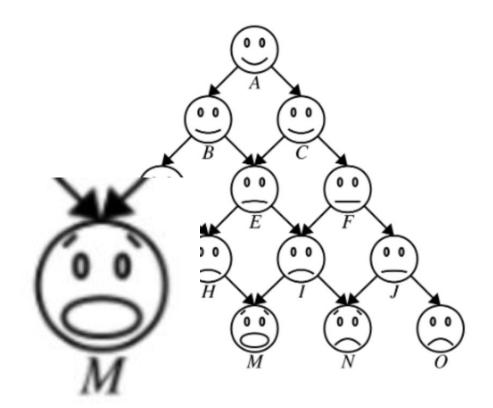
- We will now use recursion to examine why life is JUST not fair...
- Have you ever made a human pyramid with your friends, and you were placed at the bottom center?
- ...

- We will now use recursion to examine why life is JUST not fair...
- Have you ever made a human pyramid with your friends, and you were placed at the bottom center?
- ...
- OUCH!

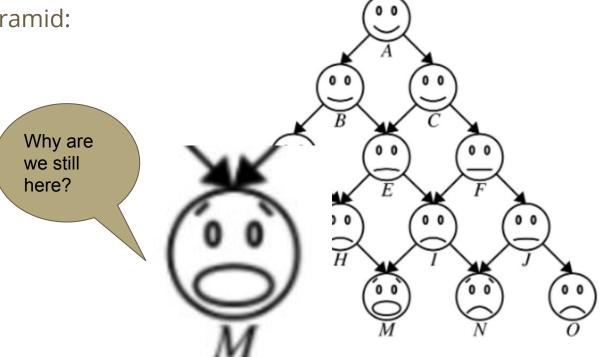
Here's a human pyramid:



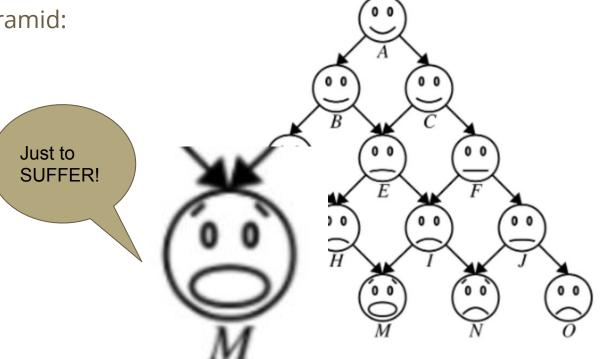
Here's a human pyramid:



• Here's a human pyramid:

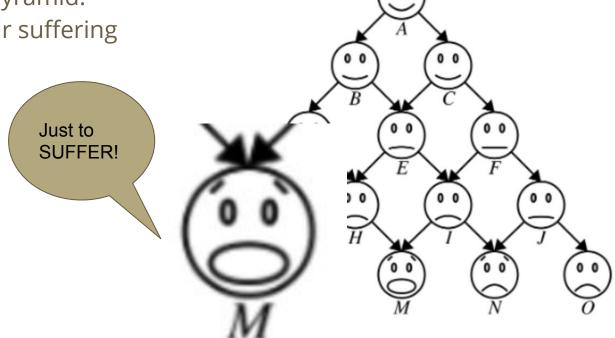


• Here's a human pyramid:



Here's a human pyramid:

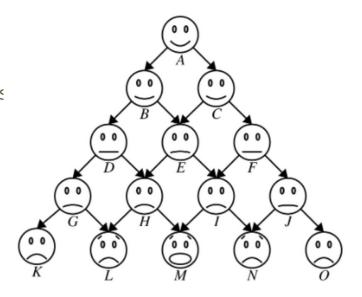
Let's quantify their suffering



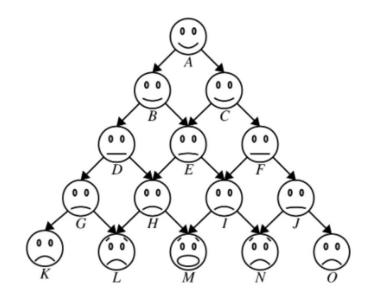


Problem set-up

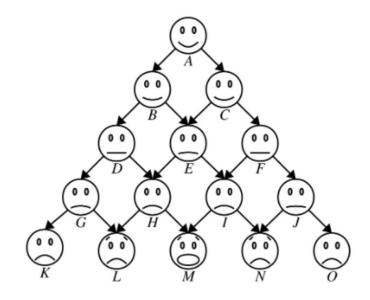
- Each person supports half the body weight of each of the people immediately above them, plus half of the weight that each of those people are supporting.
- Each person weighs 160 pounds.



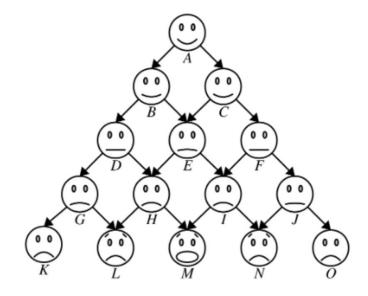
Let's try a few examples!



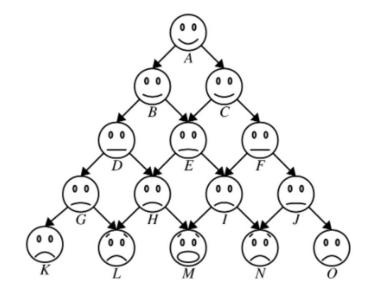
How much weight is A carrying?



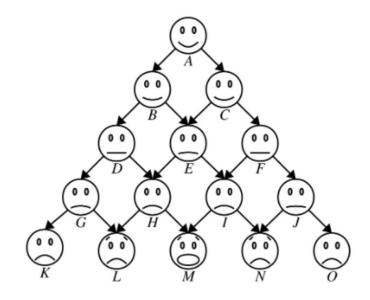
- How much weight is A carrying?
 - O lbs; no calculation needed, there are no one above them A.



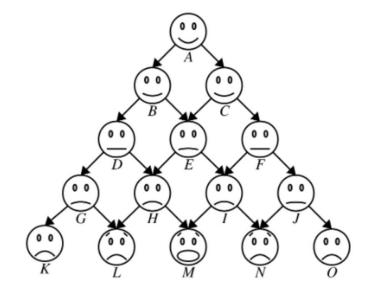
- How much weight is A carrying?
 - O lbs; no calculation needed, there are no one above them A.
 - This problem almost... trivial;)



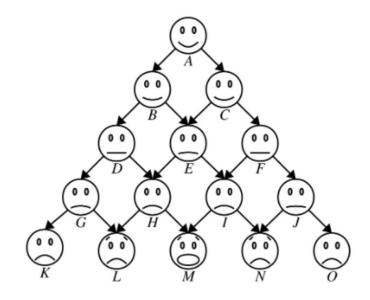
How much weight is **D** carrying?



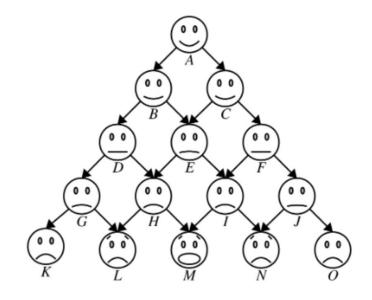
- How much weight is **D** carrying?
 - B is above D, so D carries half of B's weight.



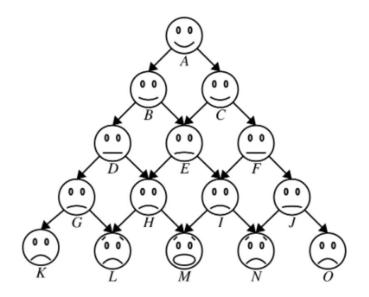
- How much weight is **D** carrying?
 - o **B** is above **D**, so **D** carries half of **B**'s weight.
 - That is 80 lbs.
 - o **D** also carries half of the weight **B** is carrying.



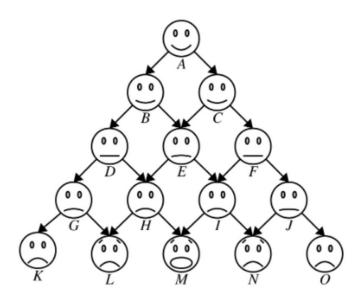
- How much weight is **D** carrying?
 - o **B** is above **D**, so **D** carries half of **B**'s weight.
 - That is 80 lbs.
 - o **D** also carries half of the weight **B** is carrying.
 - How much weight is **B** carrying?



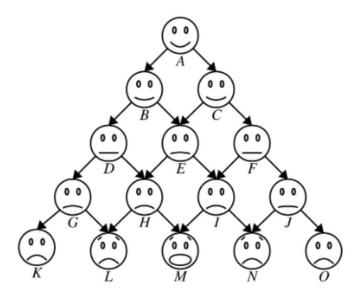
- How much weight is **D** carrying?
 - B is above D, so D carries half of B's weight.
 - That is 80 lbs.
 - D also carries half of the weight B is carrying.
 - How much weight is **B** carrying?
 - **A** is above **B**, so **B** carries half of **A**'s weight.



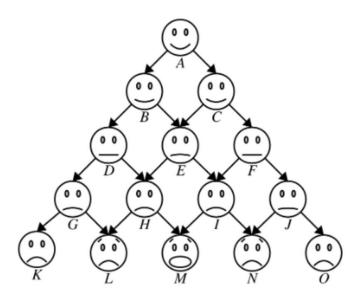
- How much weight is **D** carrying?
 - B is above D, so D carries half of B's weight.
 - That is 80 lbs.
 - D also carries half of the weight B is carrying.
 - How much weight is **B** carrying?
 - A is above B, so B carries half of A's weight.
 - That is 80 lbs. And half of that is 40 lbs.



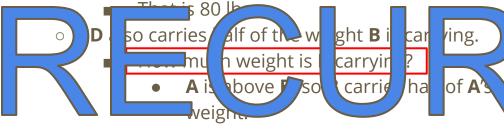
- How much weight is **D** carrying?
 - B is above D, so D carries half of B's weight.
 - That is 80 lbs.
 - D also carries half of the weight B is carrying.
 - How much weight is **B** carrying?
 - A is above B, so B carries half of A's weight.
 - That is 80 lbs. And half of that is 40 lbs.
 - In total, **D** is carrying **120** lbs.



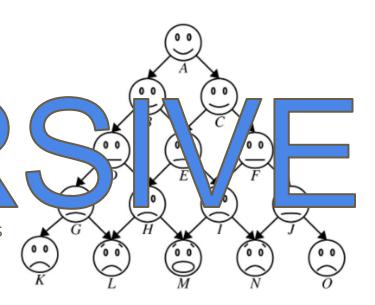
- How much weight is D carrying?
 - B is above D, so D carries half of B's weight.
 - That is 80 lbs.
 - D also carries half of the weight B is carrying.
 - How much weight is **B** carrying?
 - A is above B, so B carries half of A's weight.
 - That is 80 lbs. And half of that is 40 lbs.
 - o In total, **D** is carrying **120** lbs.



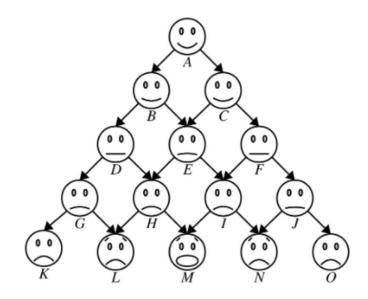
- How much weight is **D** carrying?
 - o **B** is above **D**, so **D** carries half of **B**'s weight.



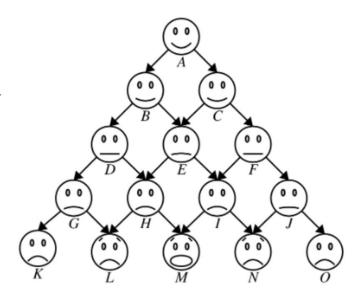
- That is 80 lbs. And half of that is 40 lbs.
- o In total, **D** is carrying **120** lbs.



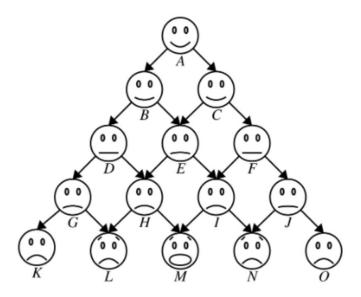
How much weight is **E** carrying?



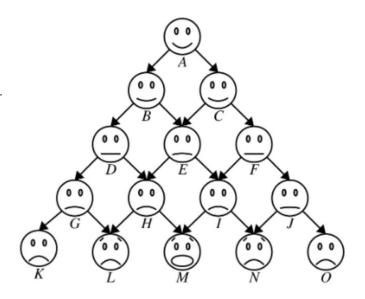
- How much weight is **E** carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.



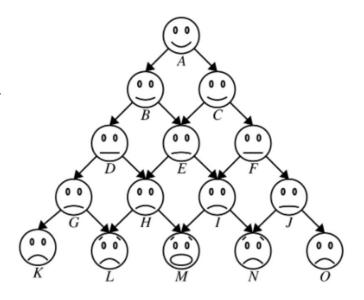
- How much weight is E carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - C is above E, so E carries half of C's weight.



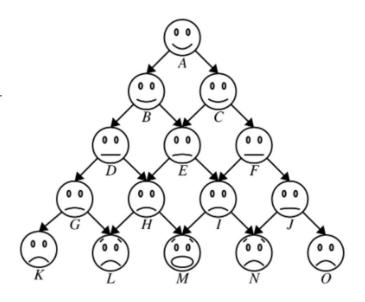
- How much weight is E carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - **C** is above **E**, so **E** carries half of **C**'s weight.
 - That is 80 lbs.



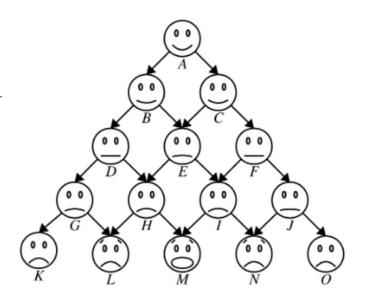
- How much weight is **E** carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - **C** is above **E**, so **E** carries half of **C**'s weight.
 - That is 80 lbs.
 - **E** also carries half of the weight **C** is carrying.



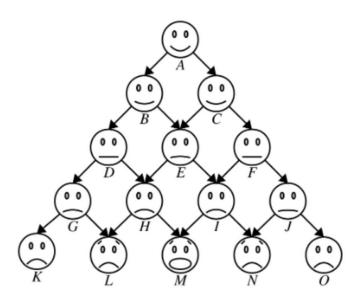
- How much weight is **E** carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - **C** is above **E**, so **E** carries half of **C**'s weight.
 - That is 80 lbs.
 - **E** also carries half of the weight **C** is carrying.
 - How much weight is **C** carrying?



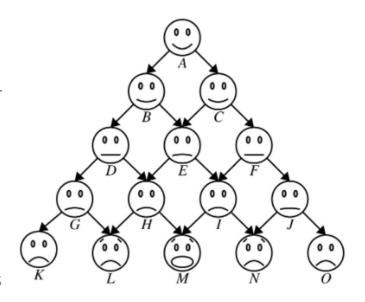
- How much weight is **E** carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - **C** is above **E**, so **E** carries half of **C**'s weight.
 - That is 80 lbs.
 - **E** also carries half of the weight **C** is carrying.
 - How much weight is **C** carrying?
 - **A** is above **C**, so **C** carries half of **A**'s weight.



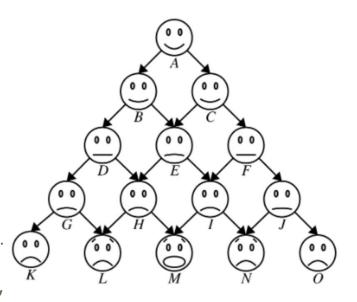
- How much weight is E carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - **C** is above **E**, so **E** carries half of **C**'s weight.
 - That is 80 lbs.
 - **E** also carries half of the weight **C** is carrying.
 - How much weight is **C** carrying?
 - A is above C, so C carries half of A's weight.
 - That is 80 lbs. And half of that is 40 lbs.



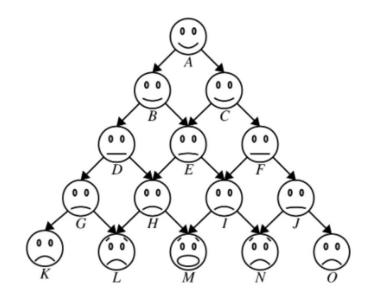
- How much weight is E carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - **C** is above **E**, so **E** carries half of **C**'s weight.
 - That is 80 lbs.
 - E also carries half of the weight C is carrying.
 - How much weight is **C** carrying?
 - A is above C, so C carries half of A's weight.
 - That is 80 lbs. And half of that is 40 lbs.
 - In total, E is carrying 240 lbs.



- How much weight is **E** carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.
 - C is above E, so E carries half of C's weight.
 - That is 80 lbs.
 - **E** also carries half of the weight **C** is carrying.
 - How much weight is **C** carrying?
 - A is above C, so C carries half of A's weight.
 - o That is 80 lbs. And half of that is 40 lbs.
 - In total, E is carrying 240 lbs.
- Despite the fact that **D** and **E** are on the same row,
 E is carrying double the weight **D** is carrying!

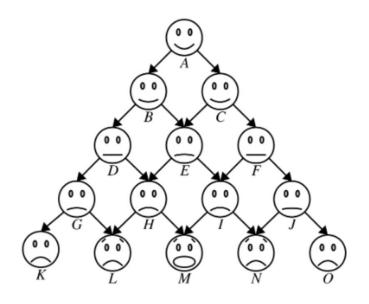


How much weight is M carrying?

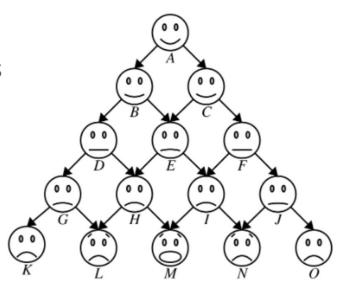


How much weight is M carrying?

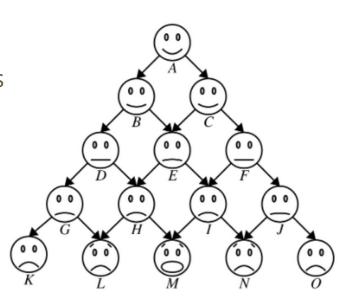




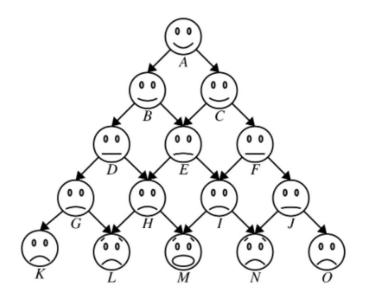
- How much weight is M carrying?
- Sorry I'm just kidding, we are not doing this LOL...



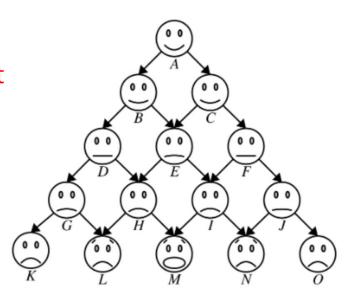
- How much weight is M carrying?
- Sorry I'm just kidding, we are not doing this LOL...
- The point is, this problem fairly computationally heavy after just several rows.



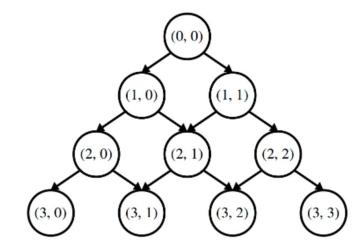
Let's get into the code!



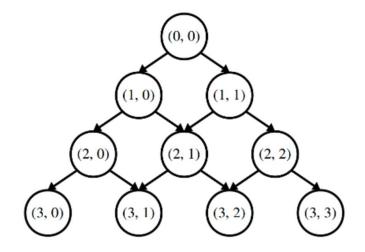
- Let's get into the code!
- double weightOnBackOf(int row, int col, int pyramidHeight);



- Let's get into the code!
- double weightOnBackOf(int row, int col, int pyramidHeight);
- Coordinate system (row, col) →

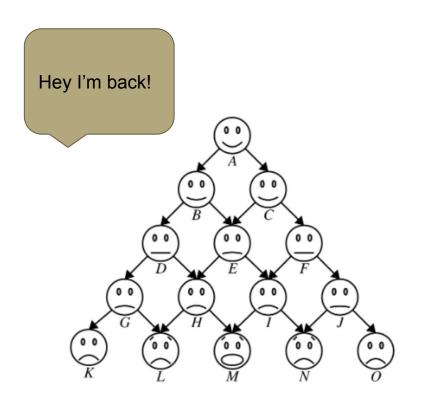


- Let's get into the code!
- double weightOnBackOf(int row, int col, int pyramidHeight);
- Coordinate system (row, col) →
- "pyramidHeight" refers to the number of rows



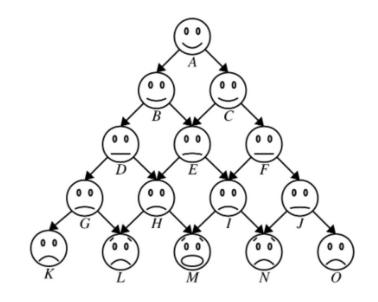
• Quiz time!

Quiz time!



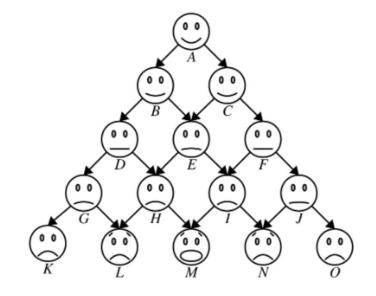
double weightOnBackOf(int row, int col, int pyramidHeight);

 What is the function call to get the weight on back of H?

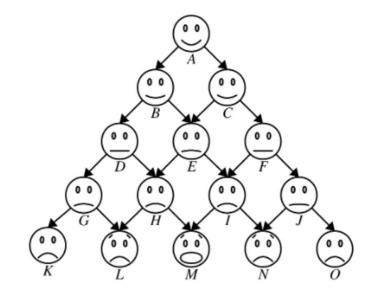


double weightOnBackOf(int row, int col, int pyramidHeight);

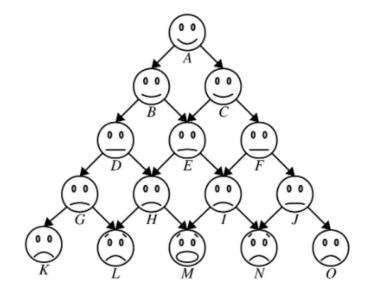
- What is the function call to get the weight on back of **H**?
 - weightOnBackOf(3, 1, 5)



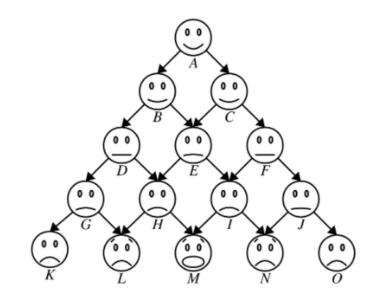
- What is the function call to get the weight on back of **H**?
 - weightOnBackOf(3, 1, 5)
- Which person is weightOnBackOf(4, 3, 5) referring to?



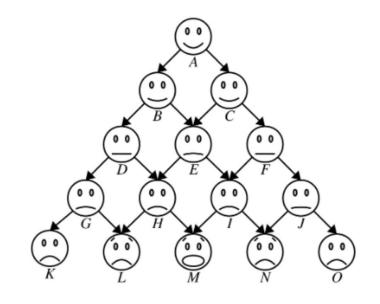
- What is the function call to get the weight on back of **H**?
 - weightOnBackOf(3, 1, 5)
- Which person is weightOnBackOf(4, 3, 5) referring to?
 - Person N



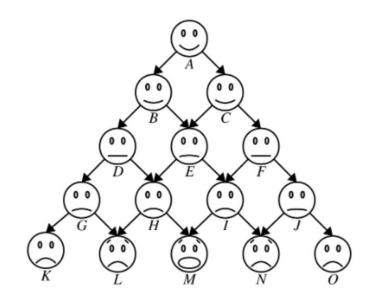
- What is the function call to get the weight on back of **H**?
 - weightOnBackOf(3, 1, 5)
- Which person is weightOnBackOf(4, 3, 5) referring to?
 - Person N
- Which person is weightOnBackOf(2, 4, 5) referring to?



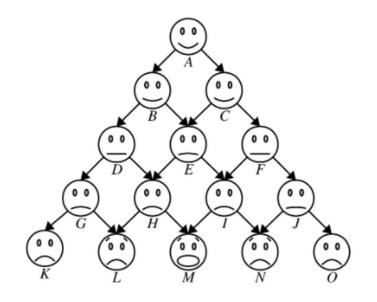
- What is the function call to get the weight on back of **H**?
 - weightOnBackOf(3, 1, 5)
- Which person is weightOnBackOf(4, 3, 5) referring to?
 - Person N
- Which person is weightOnBackOf(2, 4, 5) referring to?
 - O No one :(



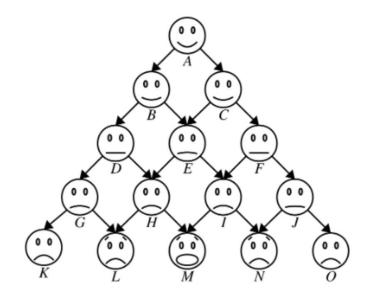
- What is the function call to get the weight on back of H?
 - weightOnBackOf(3, 1, 5)
- Which person is weightOnBackOf(4, 3, 5) referring to?
 - Person N
- Which person is weightOnBackOf(2, 4, 5) referring to?
 - No one :(
- weightOnBackOf(6, 1, 5)?



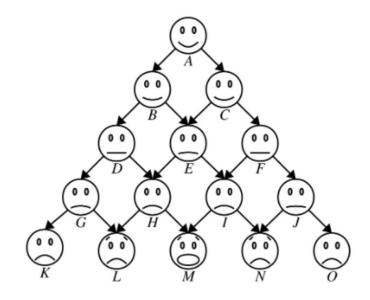
- What is the function call to get the weight on back of H?
 - weightOnBackOf(3, 1, 5)
- Which person is weightOnBackOf(4, 3, 5) referring to?
 - Person N
- Which person is weightOnBackOf(2, 4, 5) referring to?
 - o No one :(
- weightOnBackOf(6, 1, 5)?
 - o No one :(



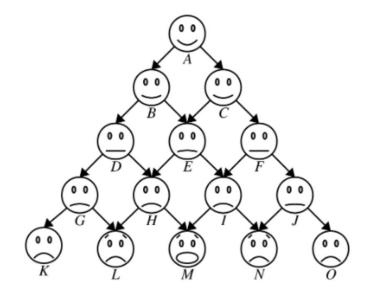
Some implementation thoughts:



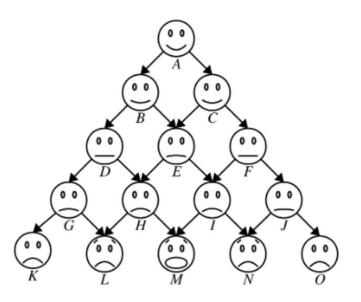
- Some implementation thoughts:
 - Throw an error for invalid function calls (think through the possible cases thoroughly).



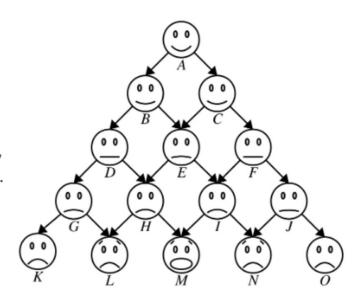
- Some implementation thoughts:
 - Throw an error for invalid function calls (think through the possible cases thoroughly).
 - The top person carries no weight.



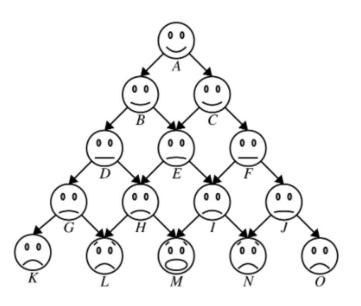
- Some implementation thoughts:
 - Throw an error for invalid function calls (think through the possible cases thoroughly).
 - The top person carries no weight.
 - Some people are directly supporting two people, some people are directly supporting one person. How should you distinguish between the two cases?



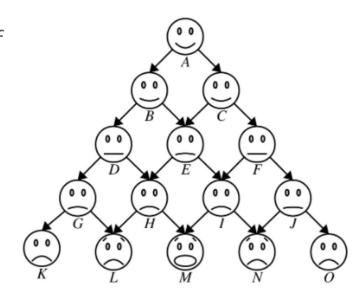
- Some implementation thoughts:
 - Throw an error for invalid function calls (think through the possible cases thoroughly).
 - The top person carries no weight.
 - Some people are directly supporting two people, some people are directly supporting one person. How should you distinguish between the two cases?
 - Remember that the function returns a **double!**



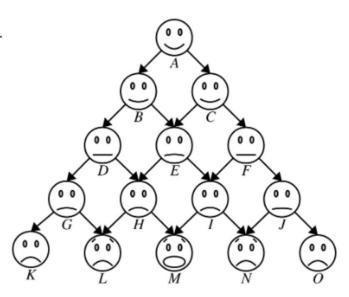
- Some implementation thoughts:
 - Throw an error for invalid function calls (think through the possible cases thoroughly).
 - The top person carries no weight.
 - Some people are directly supporting two people, some people are directly supporting one person. How should you distinguish between the two cases?
 - Remember that the function returns a **double!**
 - Test your solution before moving on!



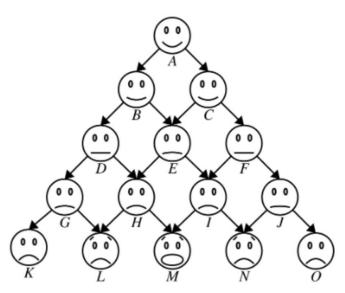
 Questions before we move on to part 2 of part 2?



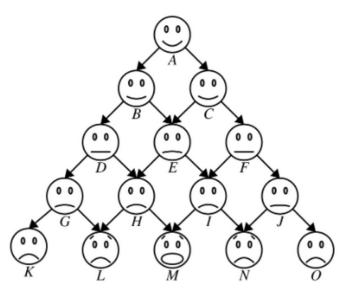
• Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.



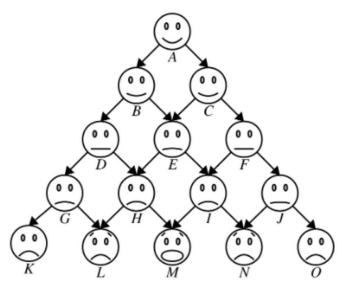
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.



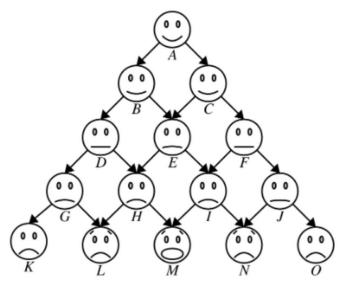
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?



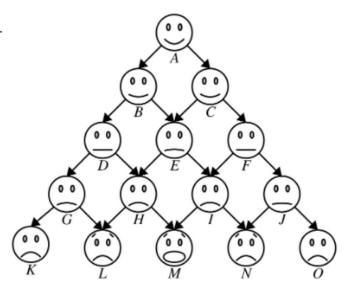
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?
 - Just once



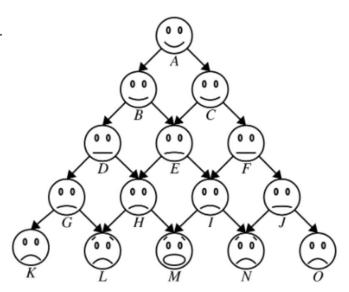
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?
 - Just once
- What about H?



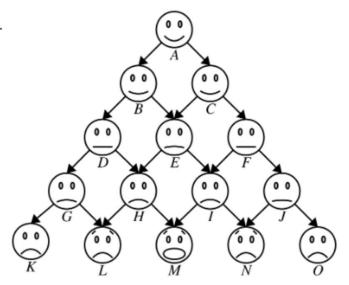
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?
 - Just once
- What about **H**?
 - Once



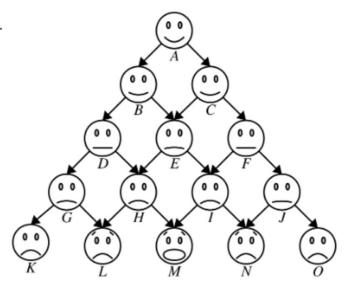
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?
 - Just once
- What about **H**?
 - o Once
- E?



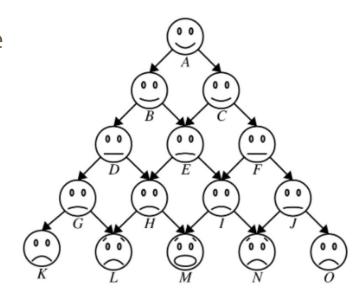
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?
 - Just once
- What about **H**?
 - o Once
- E?
 - Twice



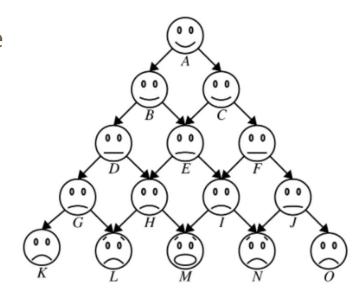
- Let's examine a <u>HUGE</u> efficiency flaw in our first implementation.
- Say we're interested in finding out how much weight M is carrying.
- How many times do we need to call the function with respect to M?
 - Just once
- What about **H**?
 - o Once
- E?
 - Twice
- And A will be called 6 times!



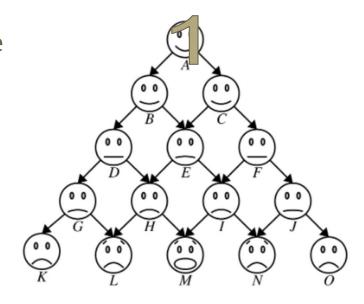
 This actually resembles a Pascal's Triangle (off topic, just slightly)



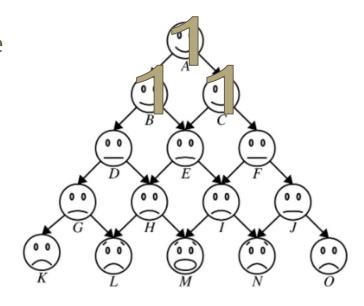
- This actually resembles a Pascal's Triangle (off topic, just slightly)
- Let the number on a person denote how many times this person needs to call A.



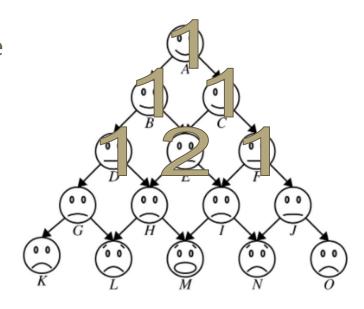
- This actually resembles a Pascal's Triangle (off topic, just slightly)
- Let the number on a person denote how many times this person needs to call A.



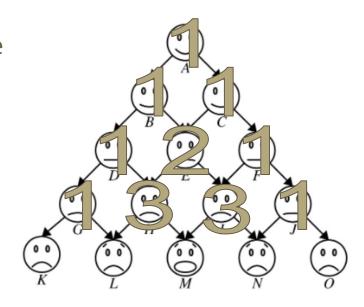
- This actually resembles a Pascal's Triangle (off topic, just slightly)
- Let the number on a person denote how many times this person needs to call A.



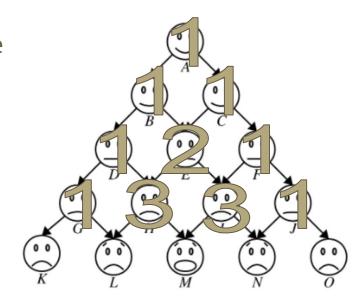
- This actually resembles a Pascal's Triangle (off topic, just slightly)
- Let the number on a person denote how many times this person needs to call A.



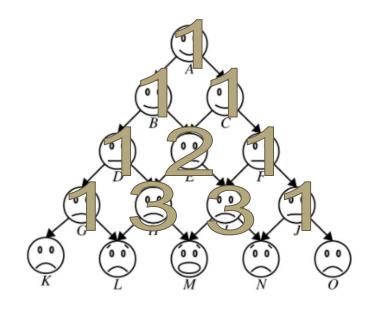
- This actually resembles a Pascal's Triangle (off topic, just slightly)
- Let the number on a person denote how many times this person needs to call A.



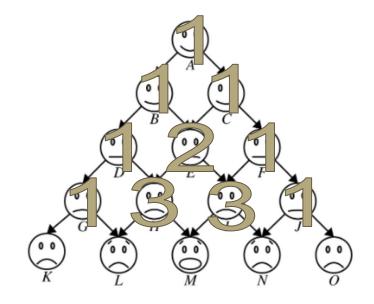
- This actually resembles a Pascal's Triangle (off topic, just slightly)
- Let the number on a person denote how many times this person needs to call A.
- You are expecting the last row filled out,
 but I'm out of animation budget: |



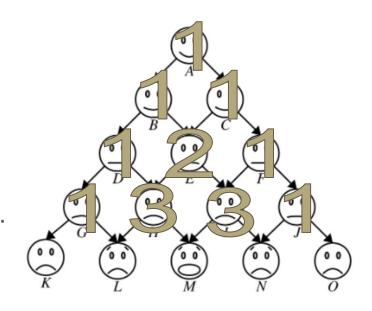
 Observation: As we get further into the recursive stack, the more copies of the SAME function calls we will be creating.



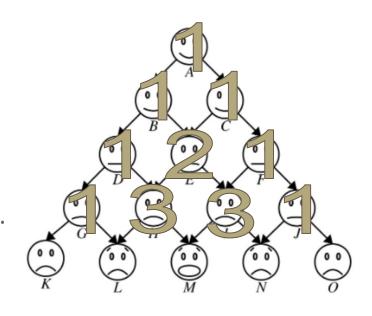
- Observation: As we get further into the recursive stack, the more copies of the SAME function calls we will be creating.
- Just the 10th row and we have surpassed
 100 in the Pascal's Triangle.



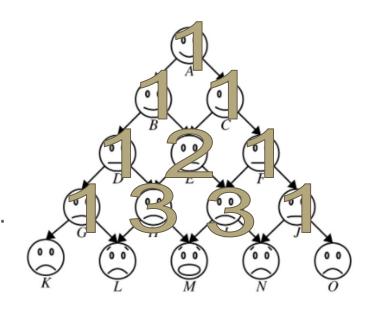
- Observation: As we get further into the recursive stack, the more copies of the SAME function calls we will be creating.
- Just the 10th row and we have surpassed
 100 in the Pascal's Triangle.
- 14th row and we have surmounted 1,000.



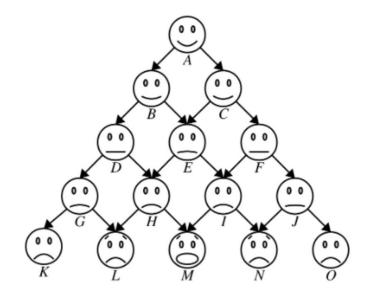
- Observation: As we get further into the recursive stack, the more copies of the SAME function calls we will be creating.
- Just the 10th row and we have surpassed
 100 in the Pascal's Triangle.
- 14th row and we have surmounted 1,000.
- 17th row > 10,000.



- Observation: As we get further into the recursive stack, the more copies of the SAME function calls we will be creating.
- Just the 10th row and we have surpassed
 100 in the Pascal's Triangle.
- 14th row and we have surmounted 1,000.
- 17th row > 10,000.
- Conclusion: This grows really really fast!

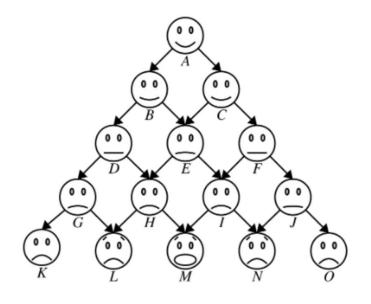


We need a solution.



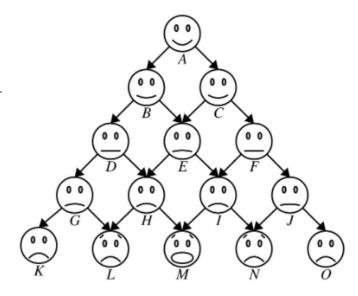
We need a solution.





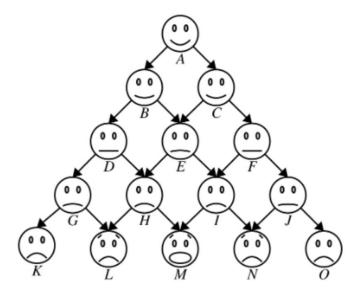
Part 2: Human Pyramids

- How much weight is **E** carrying?
 - To save some work, we know from the previous example that **B** yields 80 lbs (half of **B**'s weight) + 40 lbs (half of the weight **B** is carrying) = 120 lbs.

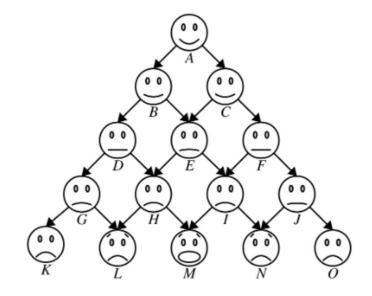


Part 2: Human Pyramids

- How much weight is **E** carrying?
 - To save some work, we know from the previous
 example that B yields 80 lbs (half of B's weight) +
 40 lbs (half of the weight B is carrying) = 120 lbs.



 Memoization: using an auxiliary table to keep track of all the recursive calls that have been made before and what value was returned for each of them.



Standard Recursion:

```
Ret function(Arg a) {
   if (base-case-holds) {
     return base-case-value;
   } else {
     do-some-work;
     return recursive-step-value;
   }
}
```

```
Ret functionRec(Arg a, Table& table) {
   if (base-case-holds) {
      return base-case-value;
   } else if (table contains a) {
      return table[a];
   } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

```
Ret functionRec(Arg a, Table& table) {
   if (base-case-holds) {
      return base-case-value;
   } else if (table contains a) {
      return table[a];
   } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
                                                    Wrapper function
   return functionRec(a, table);
```

```
Ret functionRec(Arg a, Table& table) {
   if (base-case-holds) {
      return base-case-value;
   } else if (table contains a) {
                                                   Utilizing the table if possible
     return table[a];
   } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
                                                   Wrapper function
   return functionRec(a, table);
```

```
Ret functionRec(Arg a, Table& table) {
   if (base-case-holds) {
      return base-case-value;
   } else if (table contains a) {
                                                  Utilizing the table if possible
     return table[a];
   } else {
      do-some-work;
                                                  Updating the table after computation
      table[a] = recursive-step-value
      return recursive-step-value;
Ret function(Arg a) {
   Table table;
                                                  Wrapper function
   return functionRec(a, table);
```

Note that there isn't a variable of type
 Table

```
Ret functionRec(Arg a, Table& table) {
   if (base-case-holds) {
      return base-case-value;
   } else if (table contains a) {
      return table[a];
  } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

- Note that there isn't a variable of type
 Table
 - Table is generally referring some data structure we can use to store previous computations.

```
Ret functionRec(Arg a, Table& table) {
  if (base-case-holds) {
      return base-case-value;
  } else if (table contains a) {
      return table[a];
  } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

- Note that there isn't a variable of type
 Table
 - Table is generally referring some data structure we can use to store previous computations.
 - Its type can differ from problem to problem.

```
Ret functionRec(Arg a, Table& table) {
  if (base-case-holds) {
      return base-case-value;
  } else if (table contains a) {
      return table[a];
  } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

- Note that there isn't a variable of type
 Table
 - Table is generally referring some data structure we can use to store previous computations.
 - Its type can differ from problem to problem.
- Question to ask yourself: What should the type of **Table** be such that:

```
Ret functionRec(Arg a, Table& table) {
  if (base-case-holds) {
      return base-case-value;
  } else if (table contains a) {
      return table[a];
  } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

- Note that there isn't a variable of type
 Table
 - Table is generally referring some data structure we can use to store previous computations.
 - Its type can differ from problem to problem.
- Question to ask yourself: What should the type of **Table** be such that:
 - It is easy and fast to look up necessary values.

```
Ret functionRec(Arg a, Table& table) {
  if (base-case-holds) {
      return base-case-value;
     else if (table contains a) {
      return table[a];
  } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

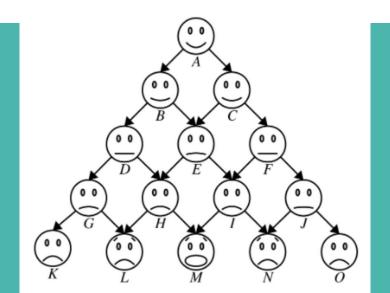
- Note that there isn't a variable of type
 Table
 - Table is generally referring some data structure we can use to store previous computations.
 - Its type can differ from problem to problem.
- Question to ask yourself: What should the type of **Table** be such that:
 - It is easy and fast to look up necessary values.
 - Can be updated during program execution.

```
Ret functionRec(Arg a, Table& table) {
  if (base-case-holds) {
      return base-case-value;
  } else if (table contains a) {
      return table[a];
  } else {
      do-some-work;
   table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

- Note that there isn't a variable of type
 Table
 - Table is generally referring some data structure we can use to store previous computations.
 - Its type can differ from problem to problem.
- Question to ask yourself: What should the type of **Table** be such that:
 - It is easy and fast to look up necessary values.
 - Can be updated during program execution.
 - Is efficient and elegant.

```
Ret functionRec(Arg a, Table& table) {
   if (base-case-holds) {
      return base-case-value;
   } else if (table contains a) {
      return table[a];
   } else {
      do-some-work;
      table[a] = recursive-step-value;
      return recursive-step-value;
Ret function(Arg a) {
    Table table;
    return functionRec(a, table);
```

Questions about Part 2?



Context Switch Time!

Definition: A *context switch* occurs when a code routine is "switched off" the CPU so that another routine can begin / resume.

• Have you ever considered an iconic phrase like:

Have you ever considered an iconic phrase like:

"To be or not to be"

Have you ever considered an iconic phrase like:

"To be or not to be"

and wondered what it would be like to capitalize every possible subset of the words?

Have you ever considered an iconic phrase like:

"To be or not to be"

and wondered what it would be like to capitalize every possible subset of the words?

Me neither!

Nonetheless, it's your job to implement the following function:

```
Set<string> allEmphasesOf(const string& sentence);
```

- Nonetheless, it's your job to implement the following function:
 Set<string> allEmphasesOf(const string& sentence);
- Given an arbitrary sentence, you'll need to return all possible combinations of emphases like so:

Nonetheless, it's your job to implement the following function:

```
Set<string> allEmphasesOf(const string& sentence);
```

 Given an arbitrary sentence, you'll need to return all possible combinations of emphases like so:

```
what are you doing?
                                            WHAT are you doing?
what are you DOING?
                                            WHAT are you DOING?
                       What are you doing?
what are YOU doing?
                                            WHAT are YOU doing?
what are YOU DOING?
                                            WHAT are YOU DOING?
what ARE you doing?
                                            WHAT ARE you doing?
what ARE you DOING?
                                            WHAT ARE you DOING?
                                            WHAT ARE YOU doing?
what ARE YOU doing?
what ARE YOU DOING?
                                            WHAT ARE YOU DOING?
```

Let's think about this critically for a second:

```
what are you doing?
```

WHAT are you doing? WHAT ARE YOU DOING?

Let's think about this critically for a second:

```
what are you doing?
                                            WHAT are you doing?
what ARF YOU DOING?
                                            WHAT ARE YOU DOING?
```

• It looks like at the upper left, *all tokens are lowercase*, and on the bottom right, all *tokens are uppercase*! Hmm, does this look like patterns you've seen before?

Granted, you might sometimes have stranger-looking questions like this:

Quoth the raven, "Nevermore."

Granted, you might sometimes have stranger-looking questions like this:

Quoth the raven, "Nevermore."

 We've provided you with the following function to help **tokenize** the given sentence.

Vector<string> tokenize(const string& sentence);

Granted, you might sometimes have stranger-looking questions like this:

Quoth the raven, "Nevermore."

 We've provided you with the following function to help **tokenize** the given sentence.

Vector<string> tokenize(const string& sentence);

That can turn the above sentence into the following vector:

Quoth the raven , "Nevermore . "

Quoth the raven , "Nevermore . "

 The good news is, you can determine whether an individual token is a word or not by checking whether the first character is alphabetical!

Quoth the raven , "Nevermore . "

- The good news is, you can determine whether an individual token is a word or not by checking whether the first character is alphabetical!
 - That might look something like this: isalpha (tokenizedString[someIndex][0])

Quoth the raven, "Nevermore."

- The good news is, you can determine whether an individual token is a word or not by checking whether the first character is alphabetical!
 - That might look something like this: isalpha(tokenizedString[someIndex][0])
 - This way you can avoid trying to capitalize / lowercase strings that aren't words! To repeat,
 you should ignore non-words!

• Some notes about this problem:

- Some notes about this problem:
 - Think long and hard about what kind of recursion you're doing here. Are you dealing with subsets? Permutations? Combinations? Something else? Be sure you can answer this question before writing code.

- Some notes about this problem:
 - Think long and hard about what kind of recursion you're doing here. Are you dealing with subsets? Permutations? Combinations? Something else? Be sure you can answer this question before writing code.
 - Helper functions are encouraged to complete this part of the assignment!

- Some notes about this problem:
 - Think long and hard about what kind of recursion you're doing here. Are you dealing with subsets? Permutations? Combinations? Something else? Be sure you can answer this question before writing code.
 - Helper functions are encouraged to complete this part of the assignment!
 - You should ignore the original casings of words, but be aware that you must BOTH toUpperCase() AND toLowerCase() words in the sentence! Students sometimes think you only need to use toUpperCase()

- Some notes about this problem:
 - Think long and hard about what kind of recursion you're doing here. Are you dealing with subsets? Permutations? Combinations? Something else? Be sure you can answer this question before writing code.
 - Helper functions are encouraged to complete this part of the assignment!
 - You should ignore the original casings of words, but be aware that you must BOTH toUpperCase() AND toLowerCase() words in the sentence! Students sometimes think you only need to use toUpperCase()
 - You shouldn't need to call the tokenize() function more than once to complete this function!

- Some notes about this problem:
 - Think long and hard about what kind of recursion you're doing here. Are you dealing with subsets? Permutations? Combinations? Something else? Be sure you can answer this question before writing code.
 - Helper functions are encouraged to complete this part of the assignment!
 - You should ignore the original casings of words, but be aware that you must BOTH toUpperCase() AND toLowerCase() words in the sentence! Students sometimes think you only need to use toUpperCase()
 - You shouldn't need to call the tokenize() function more than once to complete this function!
 - Here's how to properly use toUpperCase():

```
string str = "hello there!"
str = toUpperCase(str);
```

- Some notes about this problem:
 - Think long and hard about what kind of recursion you're doing here. Are you dealing with subsets? Permutations? Combinations? Something else? Be sure you can answer this question before writing code.
 - Helper functions are encouraged to complete this part of the assignment!
 - You should ignore the original casings of words, but be aware that you must BOTH toUpperCase() AND toLowerCase() words in the sentence! Students sometimes think you only need to use toUpperCase()
 - You shouldn't need to call the tokenize() function more than once to complete this function!
 - Here's how to properly use toUpperCase():

```
string str = "hello there!"
str = toUpperCase(str);
```

Any questions?

This is it! You've made it to the last part of the assignment!

- This is it! You've made it to the last part of the assignment!
- In this final challenge, you'll be tasked with creating a schedule for a worker that optimizes profit.

- This is it! You've made it to the last part of the assignment!
- In this final challenge, you'll be tasked with creating a schedule for a worker that optimizes profit.
 - Wait a minute, does this sound ethical?

- This is it! You've made it to the last part of the assignment!
- In this final challenge, you'll be tasked with creating a schedule for a worker that optimizes profit.
 - Wait a minute, does this sound ethical?



 Optimizing a shift schedule for profit is absolutely not ethical! Always be conscientious about what you're optimizing.

- Optimizing a shift schedule for profit is absolutely not ethical! Always be conscientious about what you're optimizing.
- Despite this... we're going to ask you to do just this in Shift Scheduling

Here's what you'll be implementing:

```
Set<Shift> highestValueScheduleFor(const Set<Shift>& shifts, int maxHours);
```

Here's what you'll be implementing:

```
Set<Shift> highestValueScheduleFor(const Set<Shift>& shifts, int maxHours);
```

Your job is to return the collection of shift's that maximizes profit given
a full collection of shifts and a maximum number of hours an individual
can work.

- Here's what you'll be implementing:
 Set<Shift> highestValueScheduleFor(const Set<Shift>& shifts, int maxHours);
- Your job is to return the collection of shift's that maximizes profit given a full collection of shifts and a maximum number of hours an individual can work.
- Here are a few functions we've written for you that use shift:

Here's how we want you to approach this problem:

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.

- Here's how we want you to approach this problem:
 - Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?
 - If this shift is invalid for either reason, discard it, and repeat the process on a new shift (does this sound self-similar?)

- Here's how we want you to approach this problem:
 - Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?
 - o If this shift is invalid for either reason, discard it, and repeat the process on a new shift (does this sound self-similar?)
 - If the shift is valid, you have **two** options:

- Here's how we want you to approach this problem:
 - Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?
 - If this shift is invalid for either reason, discard it, and repeat the process on a new shift (does this sound self-similar?)
 - If the shift is valid, you have two options:
 - See what happens if you include this shift, and continue for a new shift

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?
 - If this shift is invalid for either reason, discard it, and repeat the process on a new shift (does this sound self-similar?)
 - If the shift is valid, you have **two** options:
 - See what happens if you include this shift, and continue for a new shift
 - See what happens if you **exclude** this shift, and continue for a new shift

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?
 - o If this shift is invalid for either reason, discard it, and repeat the process on a new shift (does this sound self-similar?)
 - If the shift is valid, you have **two** options:
 - See what happens if you **include** this shift, and continue for a new shift
 - See what happens if you **exclude** this shift, and continue for a new shift
 - We want you to try **both**, and return the collection of shifts that has the **greater** aggregate value.

- Here's how we want you to approach this problem:
 - o Take some shift (any you'd like!) from the collection of given shifts.
 - Determine whether the employee can actually take this shift:
 - Does this shift's length put the worker over the max number of hours they're allowed to work?
 - Does this shift overlap with a shift the worker is already working?
 - If this shift is invalid for either reason, discard it, and repeat the process on a new shift (does this sound self-similar?)
 - If the shift is valid, you have **two** options:
 - See what happens if you **include** this shift, and continue for a new shift
 - See what happens if you **exclude** this shift, and continue for a new shift
 - We want you to try **both**, and return the collection of shifts that has the **greater** aggregate value.

Here's how we **DON'T** want you to approach this problem:

- Here's how we **DON'T** want you to approach this problem:
 - Recursively enumerate ALL possible combinations of shifts, then loop through the conglomerate and find the most valuable schedule that is feasible

- Here's how we **DON'T** want you to approach this problem:
 - Recursively enumerate ALL possible combinations of shifts, then loop through the conglomerate and find the most valuable schedule that is feasible
 - Why is this such a bad idea?

Here are a few tips about this problem:

- Here are a few tips about this problem:
 - o Follow the step-by-step approach we've given you here/on the handout!

- Here are a few tips about this problem:
 - Follow the step-by-step approach we've given you here/on the handout!
 - You shouldn't actually need to look at the internals of the shift struct! Simply use the three functions we've provided for you:)

- Here are a few tips about this problem:
 - Follow the step-by-step approach we've given you here/on the handout!
 - You shouldn't actually need to look at the internals of the shift struct! Simply use the three functions we've provided for you:)
 - An optimized collection of shifts might not use up all of the hours of a worker!

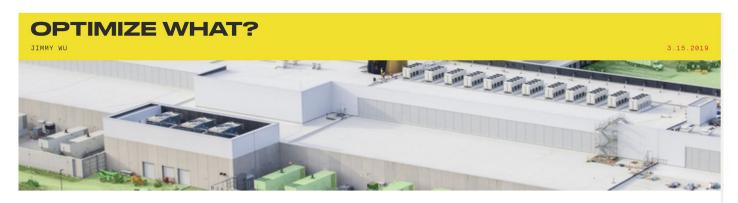
- Here are a few tips about this problem:
 - Follow the step-by-step approach we've given you here/on the handout!
 - You shouldn't actually need to look at the internals of the shift struct! Simply use the three functions we've provided for you:)
 - An optimized collection of shifts might not use up all of the hours of a worker!
 - Workers can have a maxHours value of 0, but you should raise an error() if that value is ever negative!

- Here are a few tips about this problem:
 - Follow the step-by-step approach we've given you here/on the handout!
 - You shouldn't actually need to look at the internals of the shift struct! Simply use the three functions we've provided for you:)
 - An optimized collection of shifts might not use up all of the hours of a worker!
 - Workers can have a maxHours value of 0, but you should raise an error() if that value is ever negative!
 - Remember to think long and hard about the implications of your work -- take a look at some of the schedules your algorithms produce -- you might be optimizing profit, but at what cost?

- Here are a few tips about this problem:
 - Follow the step-by-step approach we've given you here/on the handout!
 - You shouldn't actually need to look at the internals of the shift struct! Simply use the three functions we've provided for you:)
 - An optimized collection of shifts might not use up all of the hours of a worker!
 - Workers can have a maxHours value of 0, but you should raise an error() if that value is ever negative!
 - Remember to think long and hard about the implications of your work -- take a look at some of the schedules your algorithms produce -- you might be optimizing profit, but at what cost?
 - When making your recursive calls, ensure that **every** variable is being updated correctly!
 Ensure you can justify each modification you make to your parameters.

Any questions about part 4?

I'd recommend taking a look through Jimmy Wu's article "Optimize What?" It's an eye-opening piece about how a-seemingly innocuous CS education can be a *very* dangerous thing for society.



Silicon Valley is full of the stupidest geniuses you'll ever meet. The problem begins in the classrooms where computer science is taught.

Congrats!

This was a **big** assignment, but we believe in you! Remember that you have

lots of support!



When your CS instructor is teaching you about recursive functions and you think you found one

